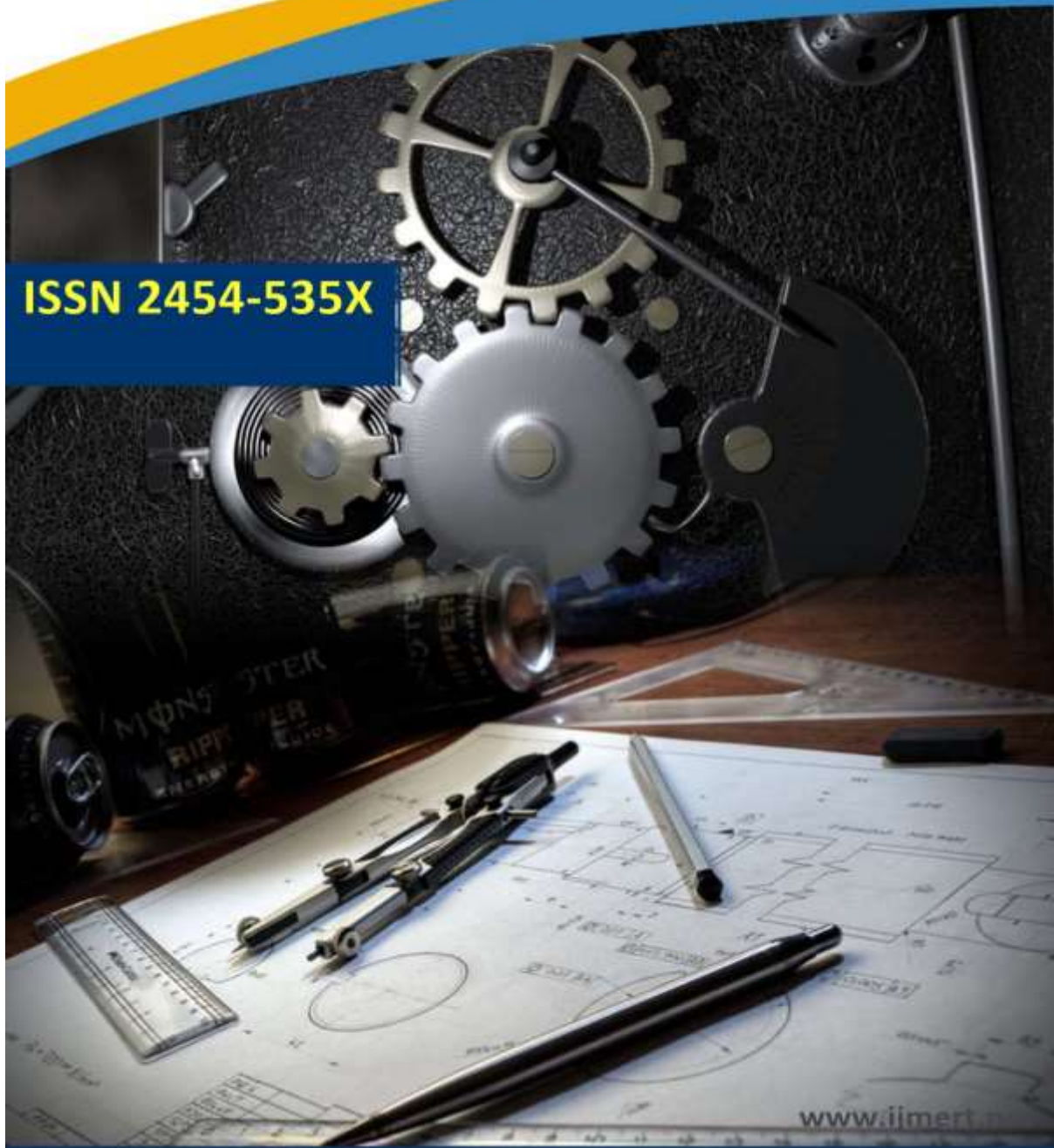




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Research on the rigidity of a three-pulley, two-universe (PUU) parallel kinematic machine

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Abstract

In this paper, we characterize the rigidity of a 3-PUU translational parallel kinematic machine (PKM). A distinct method is used to build the stiffness matrix for actuators, constraints, and leg compliance. Extreme levels of stiffness are used to assess the performance of rigidity manipulators and the implications they have on the design process. The rigidity of the 3-PUU PKM was intentionally built into its structure, which is a brilliant move. An eigenscrew decomposition of the PKM's stiffness matrix allows for the determination of the PKM's stiffness center and compliant axis, thereby offering a physical interpretation of PKM stiffness. The kinematics and mechanical rigidity of parallel manipulators

Introduction

Parallel manipulators' rising popularity [1] may be attributed to their many applications. The inherent benefits of parallel mechanisms, as well as the additional benefits in terms of manufacturing and operating costs, have led to the widespread adoption of parallel manipulators with less than six DOF in a variety of applications. The strength of parallel mechanisms is proportional to the cutting speed and effector accuracy at the end. It is important to measure the stiffness of a parallel kinematic machine (PKM) as early in the design process as feasible. Before the discovery of the 3-PUU mechanism, the concept of parallel translation was explored and studied [5, 6]. Although actuators and legs each have their own degree of compliance, very little study has been devoted to the total stiffness of the system. PKM in motion is investigated since the 3-PUU PKM stiffness model proposed for this study affects the structure's dynamics.

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Section 1.1 discusses stiffness modelling.

When a stiff body is supported by elastic elements, the relationship between force and deflection is linear [7], as described by a symmetrical 6x6 positive semidefinite matrix. An external static wrench is linked to a parallel manipulator's end-vector effector of compliant deformations through a 6x6 stiffness matrix. It is possible to use the flexibility of the individual compliant segments of six-leg parallel 6-DOF manipulators to create a simple stiffness model. Stiffness maps for two-DOF manipulators are time-consuming to generate. It is possible to simulate the stability of a PKM mounted on a tripod using computational labor [10]. Using the kinematic and static properties of all three legs, a parallel manipulator model of CaPaMan was developed in [11].

The current methods are insufficient to explain the rigidity of manipulators with less DOF. Previously, it has been recommended to use an overall Jacobian to build the stiffness matrix of a parallel manipulator [12]. A less-degrees-of-freedom (DOF) parallel manipulator's stiffness and actuation constraints may be summarized in its 6 x 6 matrix, as proposed in this study.

Stiffness assessment,

The workspace layout and the direction in which wrenches are applied establish the PKM's stiffness for a given anisotropic setup. The evaluation of whether or not the design satisfies stiffness criteria, or even performs as well as an ideal design, requires the construction and prediction of an object stiffness model. The stiffness behavior of a PKM has to be studied in a number of contexts. The scientific community has established and widely used a battery of performance indicators for measuring material stiffness. Strength may be measured with the use of stiffness matrices [8,10].

To further evaluate stiffness, one may look at the eigenvalue of the stiffness matrix for the relevant eigenvector [8,13]. Stiffness matrices with both small and large Eigenvalues have been shown to have a rigidity limit. [14] The ratio of the biggest eigenvalue to the lowest eigenvalue of the stiffness matrix may

be utilized as a predictor of stiffness values. The determinant of the stiffness matrix is the product of its eigenvalues, which is only one approach to assess it. By dividing the workspace volume by the stiffness matrix [15], one may get the stiffness of a 3-dof spherical parallel manipulator.

Off-diagonal components of the standard stiffness matrix prevent the stiffness attribute from being precisely defined in any direction. The low stiffness of the manipulator prevents its usage in applications, but the determinant or trace values are high since the trace cannot tell the two apart.

Even though the condition number indicates that the stiffness matrix has been improperly prepared for consistent manipulation, a machine tool's workspace must have a minimum stiffness level. This article use minimum and maximum stiffness values and their variants since they are employed in evaluating performance.

To learn about a PKM's spatial compliance, a stiffness model is required. Spacetime elastic behavior may have a physical basis if the stiffness matrix is decomposed into its eigenscrews. If the stiffness center and the compliant axis are present, [18] this interpretation may be made physically. When specifying a stiffness matrix, it is possible to extend the RCC (remote centre of compliance) idea to include off-diagonal blocks diagonalized at the center of stiffness. Even though a generic stiffness matrix's normal form is not diagonal, it is still feasible to isolate rotation and translation. In robotics, it serves as both a torsion and linear spring. When applied to the axis of a compliant system, linear and rotational deformation are parallel. This is always the case, regardless of how rigorous the system is.

In Section 2, we provide 3-PUU PKM, and in Section 3, we detail a new method for calculating the stiffness matrix. Shock indices, which are mentioned in Section 4, may be used to forecast the durability of a product's construction. The discussion is concluded in the fifth section.

Kinematic description

Figure 1 depicts the CAD model of a 3-PUU PKM, whereas Figure 2 shows the schematic design. A movable platform, a stationary base, and three arms with the same kinematic framework make up the manipulator. Lead screw linear actuators are used to drive (U) joints sequentially. Because each U joint is made up of two revolute (R) joints that meet at an angle, each limb may move like a Chain of motion PRRRR. Only translational movements can be achieved using a 3-PUU mechanism.

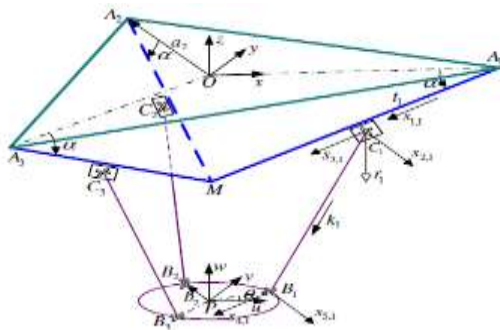


Fig. 2. Schematic representation of a 3-PUU PKM.

For each chain, the initial and last revolute joints are parallel, and the two intermediate joint axes are also parallel. Figure 2 shows the fixed Cartesian reference frame (Ox,y,z) we'll be using for this inquiry. The permanent base of the platform and the movable frame of the mobile platform. Triangle DB1A2B3 and triangle DB1B2B3 intersecting The x and u axes should be aligned to make things easy. OA1 is used to designate the x-axis. Oai and OAi are the vectors' angles to each other PBi "1; 2; 3" is a novel way of putting it. Angle h, therefore, is the angle formed by a moving platform and a stationary base. On one of its three tracks, AiM crosses across. The x-y plane has three points where circles of the same radius intersect: A1, A2, and A3, as well as M, where a third circle of the same radius crosses. Circles B1, B2, and B3 are the intersection locations of the three legs CiBi with lengths l in the U-V plane. Its circumference is b Angle an is defined as the angle of motion of the actuators from the base to the rails AiM. Perspective. To guarantee that the manipulator has a symmetric workspace, DA1A2A3 and DB1B2B3 must be used. Equilateral triangles are being distributed. Leg CiBi represents the actuator's linear displacement and its rotation. An indicator of the unit vector should be shown on the AiM rail. Make sure ai gets a quarter of OAi, too One-eighth PBI is an alternative. For every time, there is a four-fold multiplier. Vector-loop analysis may be used to address both forward and backward motion issues. Closed-form solutions may exist. Solutions to inverted kinematics may be summed up as follows: As a result of this data, the 3-PUU PKM's workspace is now revealed.

$$d_i = l_i q_i - \sqrt{(l_i^2 - c_i^2 q_i^2 + l_i^2)} \quad (1)$$

$$\text{where } q_i = p + b_i - a_i \text{ for } i = 1, 2, \text{ and } 3.$$

Stiffness matrix generation Jacobian matrix derivation

The Jacobian matrix of a parallel manipulator may be derived using reciprocal screw theory [12]. The mobile platform's twist may be described as T 14 12tT xTT in Plu cker axis coordinates, with t and x designating the vectors for linear and angular velocities,

$$\mathbf{T} = \hat{d}_1 \hat{T}_1 + \hat{d}_2 \hat{T}_2 + \hat{d}_3 \hat{T}_3 + \hat{d}_4 \hat{T}_4 + \hat{d}_5 \hat{T}_5 \quad (2)$$



A unit screw (in Plucker coordinates) is connected with each of the joints of the leg in which the intensity is equal to or greater than $\frac{1}{h_j}$, where I is 1, 2, or 3.

$$\hat{I}_1 = \begin{bmatrix} s_{11} \\ 0 \\ 0 \end{bmatrix}, \quad \hat{I}_2 = \begin{bmatrix} c_2 \times s_{21} \\ s_{21} \\ 0 \end{bmatrix}, \quad \hat{I}_3 = \begin{bmatrix} c_3 \times s_{31} \\ s_{31} \\ 0 \end{bmatrix}, \quad \hat{I}_4 = \begin{bmatrix} h_4 \times s_{41} \\ s_{41} \\ 0 \end{bmatrix}, \quad \hat{I}_5 = \begin{bmatrix} h_5 \times s_{51} \\ s_{51} \\ 0 \end{bmatrix}$$

The following equations are used to determine s_{ji} . For the 3-PUU mechanism's joint axis, the translational PKM has to fulfil criteria s_{ji} ; First, a ray coordinate of one screw t_{ji} , which is reciprocal to all other screws t_{ci} of the i th joint. Secondly, a ray coordinate A 1-system is a screw with an infinite pitch that is oriented perpendicular to the limb. The articulation of a U-joint is divided into two axes:

$$\hat{t}_{ji} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad (3)$$

Eq. (2) may be constructed into a matrix form by taking the product of both sides of the equation with t_{ji} .

$$J_i T = 0, \quad (4)$$

where

$$J_i = \begin{bmatrix} 0 & s_{11}^T \\ 0 & s_{21}^T \\ 0 & s_{31}^T \end{bmatrix}_{1 \times 2}, \quad (5)$$

is referred to as the Jacobian principle of constraint. The mobile platform's 3-DOF mobility is restricted by the combination of the limitations in each row of J_c . The unique solution to Eq. (4) if r_i is: $x \neq 0$. This system contains the, Screw t_{ji} had already figured it out. All the passive joint screws of the extra basis screw t_{ji} are reciprocal zero pitch screw may be distinguished along the path of the two U joints, i.e.

$$\hat{t}_{ji} = \begin{bmatrix} h_i \\ h_i \times k_i \end{bmatrix}, \quad (6)$$

Similarly, taking the product of both sides of Eq. (2) with \hat{t}_{ji} , leads to a matrix-form result:

$$J_i T = 0, \quad (7)$$

where $\dot{q} = [\dot{d}_1 \ \dot{d}_2 \ \dot{d}_3]^T$ denotes the actuated joint rate and

$$J_i = \begin{bmatrix} \frac{k_i^T}{k_i^T n_{1j}} & \frac{\partial n_{1j}}{\partial d_1} \\ \frac{k_i^T}{k_i^T n_{2j}} & \frac{\partial n_{2j}}{\partial d_2} \\ \frac{k_i^T}{k_i^T n_{3j}} & \frac{\partial n_{3j}}{\partial d_3} \end{bmatrix}_{3 \times 3}, \quad (8)$$

it's known as the Jacobian of motions. J_i 's units demonstrate following talks. As a result, in order to

construct a stiffness matrix, the Jacobian matrix units must be homogenised. Invariant to the length unit selected, the performance index The dimensionally homogenous J_c is dimensionless. It is possible to attain the Jacobian of actuations

$$J_{da} = J_d W, \quad (9)$$

with $W = \text{diag}[1, 1, 1, \frac{1}{b}, \frac{1}{b}, \frac{1}{b}]$, where the mobile platform radius b is chosen as the characteristic length to homogenize the dimension of the Jacobian matrix.

Combining Eqs. (4) and (7) allows the generation of

$$\dot{q}_0 = J T, \quad (10)$$

where $\dot{q}_0 = [\dot{d}_1 \ \dot{d}_2 \ \dot{d}_3 \ 0 \ 0 \ 0]^T$ is the extended joint rate, and

$$J = \begin{bmatrix} J_{da} \\ J_c \end{bmatrix}_{6 \times 6}, \quad (11)$$

is called the overall Jacobian of a 3-PUU PKM, which is homogeneous in terms of units.

Stiffness modelling is discussed in section

Three constraint couples are exerted on the movable platform by the wrench system that is the reciprocal screw system with infinite pitch and by the reciprocal screw system with zero pitch. Three forces are applied to the movable platform via the screw system of actuation. the limbs. In other words, each leg is subjected to one and a half times its own weight in a certain direction. Considering the premise Infinite rigidity of the U joints and mobile platform and the compliance of actuators and legs are the only constraints may be deduced in this manner.

Control of actuators affects compliance

To move a lead screw, the torque must be transmitted between the i th nut and the linear displacement may be estimated as a function of time

$$f_i = \frac{2\tau_i}{\mu_i d_i} \quad \text{and} \quad \Delta_i = \frac{P_i}{K_{ai}}, \quad (12)$$

Assume that μ_i is the friction coefficient of the i th actuator, τ_i is its torsional stiffness, and d_i is its pitch diameter. According to Eq. (12), one can calculate the linear driving device's compliance:

$$C_i = \frac{\Delta_i}{f_i} = \frac{K_{ai} d_i}{2\tau_i}, \quad (13)$$

As a result, the projection of compliance in the corresponding leg's direction may be deduced as a function of the i th actuator.

$$C_{li}^k = k_{li}^T s_{li} C_i \quad (14)$$

Legs-based compliance

Transverse compliance is equal to the i th leg's $C_{kl,i}$, whereas longitudinal compliance is the same. There is an elastic deformation of the i th leg because of a constraint force F_{ki} and a constraint couple M_{ri} perpendicular to the limb's universal joint. This means that the elastic deformations may be represented as follows:

$$\Delta l_i = C_{li}^k F_i^k = \frac{l_i}{AE} F_i^k \quad (15)$$

$$\Delta \theta_i = C_{li}^r M_i^r = \frac{l_i}{GJ} k_{li}^T r_i M_i^r \quad (16)$$

There are two legs, each with a length of l and a cross-sectional area of A , and each leg has a modulus of elasticity E and G , respectively. Eqs. (15) and (16) may then be used to generate $C_{kl,i}$ and $C_{kh,i}$.

The stiffness model

Constraints' and actuators' stiffnesses may be calculated using the inverse connection between stiffness and compliance

$$K_{a,i} = C_{a,i}^{-1} = (C_{a,i}^k + C_{l,i}^k)^{-1},$$

$$K_{c,i} = C_{c,i}^{-1} = (C_{\theta,i}^r)^{-1}$$

for $i = 1, 2$, and 3 .

Consider that three linear springs are used to link the movable platform to the stationary base, and three rotating springs are used as well, as shown in Fig.

Stiffness matrix determination

Suppose an external wrench w is applied to the movable platform in the form of the Plucker ray coordinate, where force is denoted by the notation F , torque is denoted by the notation M , and so on. The response forces/torques of the actuators and restraints, respectively, may be represented by the s_a and s_c symbols. Reaction forces/torques exerted by actuators and restraints, i.e., the external wrench is balanced in the absence of gravity

$$w = J_a^T r_a + J_c^T r_c \quad (18)$$

where the reaction forces/moments can be expressed as

$$r_a = K_a \Delta q_a \quad (19a)$$

$$r_c = K_c \Delta q_c \quad (19b)$$

the matrices are $v_{14} \text{diag} 1-2K_a; 1; K_a; 2-3$ and v_c , respectively, which represent the displacements of actuators and restrictions, respectively, in the form of D_{qa} and D_{qc} . It is also possible to calculate the displacements of translation and rotation of the movable platform with respect to the three reference axes by using the formula: $dx \ 14 \ 12D_x \ Dy \ DzT; dh \ 14 \ 12D_{hx} \ Dh_y \ Dh_zT$. Then, by ignoring the gravitational impact, the formation of virtual labour is possible.

$$w^T \Delta X = r_a^T \Delta q_a + r_c^T \Delta q_c \quad (20)$$

where $\Delta X = [\Delta x^T \ \Delta \theta^T]^T$ denotes the mobile platform's twist deformation in the axis coordinate.

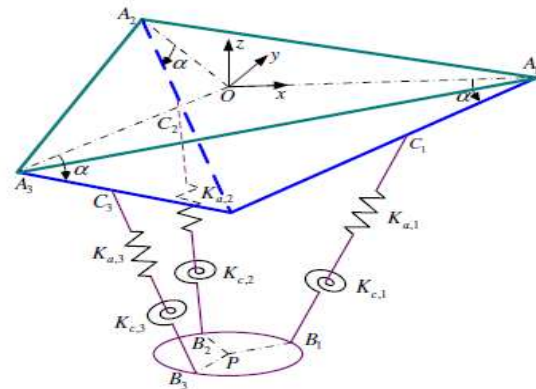


Fig. 3. Stiffness model of a 3-PUU PKM.

A careful analysis of Eqs. (18)–(20) at the same time, leads to the expression of

$$w = K \Delta X \quad (21)$$

in where K is defined as the 6-by-6 overall stiffness matrix of a 3-PUU PKM, encompassing the influence of actuators and restrictions, with the 6-by-6 diagonal matrix $v_{14} \text{diag} 12v_a \ v_c$. Where

Evaluation of the 3-PUU PKM's stiffness

As can be seen in Table 1, the 3-PUU PKM's design parameters aim to strike a balance between the

overall workspace's global dexterity index and the space utility ratio index, which measures the workspace's volume in relation to the robot's physical size [6]. In addition, the U joints' cone angle restrictions are 20, and the P joints' motion range limits are D0.1 m. The manipulator's accessible workspace is constructed as illustrated in Fig. 4 using a numerical search approach described in [19]. Moreover, Table 2 details the design's physical properties (3-PUU PKM). Di 14 0 -i 14 1; 2; 3 is the home position of the mobile platform in the case of mid-stroke linear actuators, in which the stiffness matrix is derived as follows:

Table 1

Architectural parameters of a 3-PUU PKM

Parameter	Value
a	0.3 m
b	0.1 m
l	0.3 m
α	45.0°
θ	0.0°

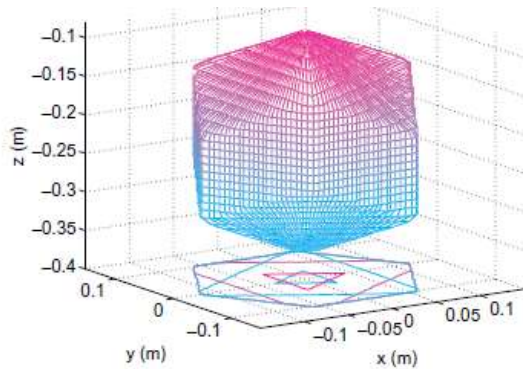


Fig. 4. Reachable workspace of a 3-PUU PKM.

Table 2

Physical parameters of a 3-PUU PKM

Parameter	Value	Parameter	Value
K_{u1}	$1.45 \times 10^6 \text{ N/m/rad}$	E	$2.05 \times 10^{11} \text{ N/m}^2$
K_{u2}	0.25	G	$7.95 \times 10^{10} \text{ N/m}^2$
\bar{L}_1	30 mm	A	$2.01 \times 10^{-4} \text{ m}^2$
f	3 mm	I_p	$3.22 \times 10^{-8} \text{ m}^4$

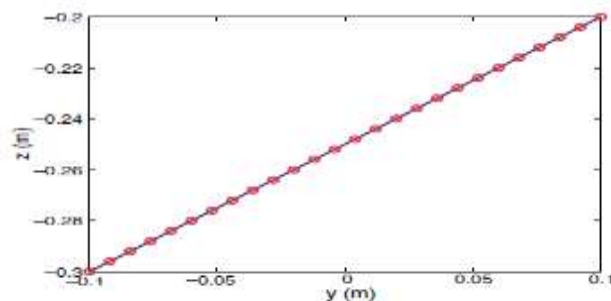
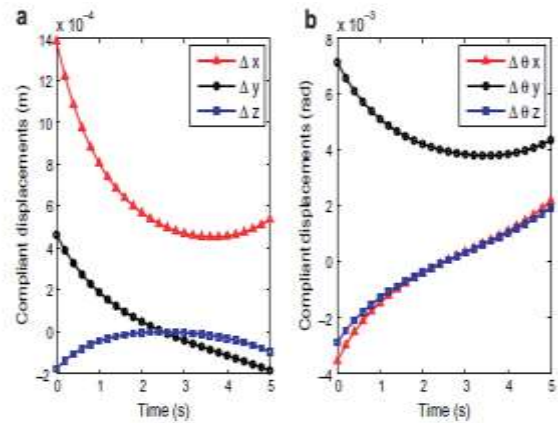
Fig. 5. Trajectory of the mobile platform in a plane of $x = 0 \text{ m}$.

Fig. 6. The compliant displacements of (a) translations and (b) rotations of the mobile platform.

$$\mathbf{k}^0 = \begin{bmatrix} 9.0891 & 0 & 0 & 0 & -1.0162 & 0 \\ 0 & 9.0891 & 0 & 1.0162 & 0 & 0 \\ 0 & 0 & 22.7228 & 0 & 0 & 0 \\ 0 & 1.0162 & 0 & 0.1137 & 0 & 0 \\ -1.0162 & 0 & 0 & 0 & 0.1137 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0001 \end{bmatrix} \times 10^7,$$

Words N/m are used in this context to describe the phrases K0 11g; K113g; and N/rad to describe the phrases N/mg; K015g; and N/mg. PKM's movable platform may be utilised to calculate the DX's compliant displacement in light of Equation (21). As seen in FIG. 6, the platform moves at a constant speed while being exposed to a static external force of 20 N. It was found that the linear compliant displacement along the x-axis was 1.4 mm, as predicted. In addition to this, the y-axis rotation is the most rotary-compliant displacement of them all.

Stiffness assessment

Manipulation of PKM requires a more rigid workspace than some fixed limit. By applying classical eigenvalue decomposition, one may get an overview of the stiffness levels in the workspace by finding the smallest and largest stiffness eigenvalues. Quantitative analysis was performed on the overall stiffness of the PKM workspace. Essential steps in this method include dividing the volume V in cartesian coordinates and evaluating the various portions to see whether they belong in the workspace. The accuracy requirements for sample size may be found here. Verification is achieved making use of mechanical joint motion constraints and inverse kinematic solutions. The elements of a stiffness matrix that are contained inside a certain workspace

may be found by decomposing the matrix. The minimum and maximum stiffness values for the workspace are determined by comparing the values at the lowest and highest points for each sample. The ease with which it may be implemented in code has led to its widespread adoption. [20] It's possible that a computer round-off analysis technique may be useful for designing a Gough-type parallel manipulator. Moreover, it may be used to generate and evaluate two 3-degrees-of-freedom PKMs [3].

The stiffness at $z = 0.242$ m (the height at which the home position is maintained) is shown in Figure 7. Three P joints, each with 120 degrees of x-y rotation, are visible in the viewing area. As a manipulator approaches the edge of the working area, both its minimum and maximum stiffness increase. The PKM's weak stiffness qualities become apparent when it is placed beyond the feasible workspace. Leaving it as it is makes the most sense. The PKM activities and metrics serve as the basis for defining this subworkspace. The platform's home position is specified as the center of a cubical work area with a 0.01 m edge length. Adjusting the kinematic parameters allows for the study of stiffness. The rigidity of this working area may be measured down to a diameter of 0.002 mm. Consistent with the 3-PUU PKM's design specifications, a little amount of variation in stiffness is seen in Figs. 8a-d.

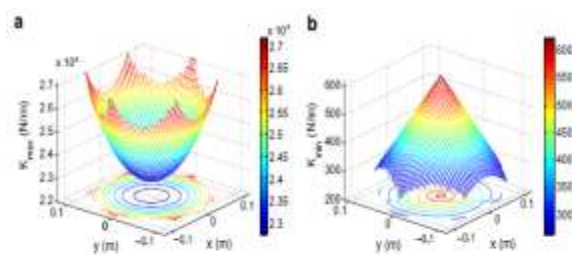


Fig. 7. The distribution for (a) minimum and (b) maximum stiffness in a plane of $z = 0.224$ m.

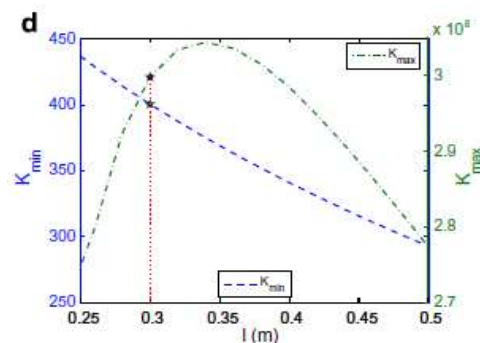
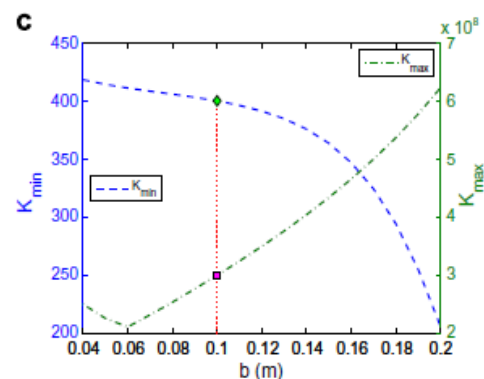
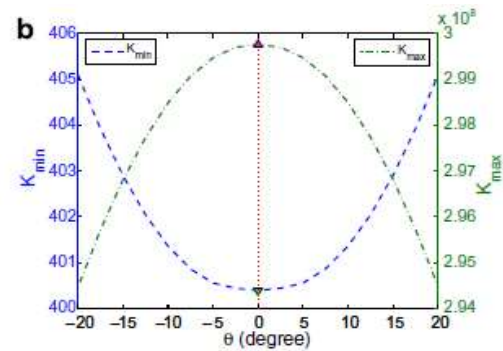
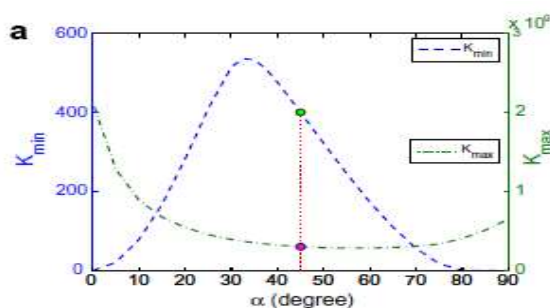


Fig. 8. Global stiffness index versus design parameters of (a) actuators layout angle, (b) twist angle, (c) mobile platform size, and (d) the leg length.

It's important to think about how stiff the manipulator is. Minimum stiffness seems to be greatest between 0 and 35 degrees, while maximum stiffness appears to be least around 60 degrees, both relative to 90 degrees. As the moveable platform size increases from 0.25 meters to 0.50 meters, the minimum and maximum stiffness values both decrease for h 14 0 twist angles, whereas the maximum stiffness increases. The minimum and maximum stiffness values for these twist angles are the lowest possible. Figure 8 shows the manipulator's stiffness at various positions. This diagram shows that the maximum and minimum stiffness requirements for agility and productivity in the workplace are not reached. The efficiency with which the PKM's architectural optimization is serving machine tool tasks may be analyzed using stiffness indices, which vary with the tasks at hand.

Stiffness interpretation via eigenscrew decomposition

To discover out how stiff the structure is, we'll do an eigenscrew matrix decomposition. Twists are represented in the axis coordinate system, while wrenches are represented in the ray coordinate system. If you want to acquire useful results from the stiffness matrix eigenscrew problem, you must construct it consistently. Some scenarios necessitate the usage of ray or axis screw-based coordinates. It also insures that the results are not depending on the coordinate frame and that the units are preserved as they should be. The results won't hold up without this step, so it's of no practical relevance. The bD matrix may be used to transition between two separate kinds of coordinate systems.

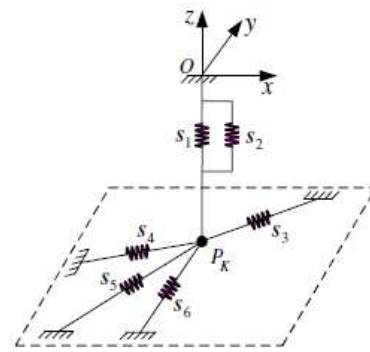


Fig. 9. The physical interpretation of the stiffness of a 3-PUU PKM.

In terms of relevance, the number 2 is important. Therefore, the spring constant k , the helical joint pitch p , and the geometrical connection parameters n and r define the spring properties of each screw spring. No doubt, the first two springs are perpendicular to one another and in the same plane as one another, while the last four are perpendicular to one another and in the "z-plane." The centre of stiffness, where rotations and translations may be decoupled to the maximum degree feasible, is represented by six springs linked at a single point.

Compliant axis determination

[18] In order to produce a compliant axis, the linear deformation must be parallel to the rotational deformation surrounding it. Only a compliant axis can address the eigenscrew issue. There must be two collinear screws with equal stiffness and opposing signs in order for a compliant shaft to operate. The two collinear eigenscrews define the compliant axis.

Table 3

Spring constant and geometrical connection for each screw spring

Spring	$k \times 10^{-4}$	\mathbf{a}^T	\mathbf{r}^T	p
s_1	1.1361	$[0, 0, 1]$	$[0, 0, 0]$	-0.0140
s_2	1.1361	$[0, 0, 1]$	$[0, 0, 0]$	0.0140
s_3	0.4545	$[0.8800, 0.4748, 0]$	$[0, 0, -0.1110]$	-0.0166
s_4	0.4545	$[-0.0262, -0.5634, 0]$	$[0, 0, -0.1110]$	0.0166
s_5	0.4545	$[-0.0236, -0.9987, 0]$	$[0, 0, -0.1110]$	-0.0166
s_6	0.4545	$[-0.0247, -0.9987, 0]$	$[0, 0, -0.1110]$	0.0166



Conclusions

Jacobian global effects of actuation and limitation are taken into consideration thanks to the use of the reciprocal screw theory in its construction. The stiffness of the manipulator is also modeled, including the actuators and the legs. Using the least and greatest eigenvalues of the stiffness matrix in a cubic form, the PKM stiffness may be evaluated. In this study, we discuss how the stiffness of a building's 3-PUU PKM is affected by design choices. The best method for understanding the PKM's malleable behavior is to deconstruct the stiffness matrix using eigenscrews. The stiffness of a body may be measured by suspending it from a series of screw springs. With the PKM, the stiffness along the z-axis is increased since the rigidity center and compliant axis are always oriented in the same direction. The physical interpretation of PKM stiffness, the measurement of PKM stiffness using architectural characteristics, and the modeling of 3-PUU PKM stiffness have all advanced significantly. The given analytic methods may be used to mimic other types of parallel manipulators. The 3-PUU PKM's stiffness characteristics may serve as a benchmark for future building designs. After the PKM is built and fabricated, experimental validation of the stiffness studies is necessary.

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